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# Harmonic tensions in "limited approximations" by Georg Friedrich Haas

Georg Friedrich Haas, a composer known for his use of microtonality, is often associated with spectralism, but he is also heavily influenced by other microtonal traditions, as well as 12-tone atonal styles. In whatever style he is writing, Haas regularly uses equal divisions of the traditional semitone, such as quarter- and sixth-tones, to approximate overtone spectra. The title for his work, *limited approximations,* is an expression of the necessity to temper even very finely tuned systems of pitch.

This 30-minute work from 2010 is subtitled "concerto for six pianos in twelfth-tones and orchestra." By retuning five of the pianos, a complete scale with seventy-two equal divisions of the octave (72EDO) is accessible. The tuning of the pianos is laid out in the score as follows:

1. Klavier: 1/6-Ton höher [ossia: 1/3-Ton tiefer]
2. Klavier: 1/12-Ton höher
3. Kalvier: Normalstimmung
4. Klavier: 1/12-Ton tiefer
5. Kalvier: 1/6-Ton tiefer
6. Kavlier: 1/4-Ton tiefer[1]

The incredible specificity of pitches made available through this 72EDO arrangement is used in two main ways throughout the work: first, to approximate, with considerable accuracy, the partials of the harmonic series to create overtone chords, and

[1] Haas (2010a)

second, to inflect or ornament pitches from the 12-tone equal tempered scale (12EDO). Much of the dramatic tension in the work is a result of the juxtaposition and combination of different approaches to the use of microtones and it is useful to trace the influence that two disparate styles have had on "limited approximations": spectralism and the Second Viennese School.

## Harmonic Materials

It is common to group all music that uses the harmonic spectrum as source material into the single category of 'spectralism,' but this is especially problematic with Haas who objects to the label, writing: "yes, I use overtone spectra. But I would protest against being called a 'spectralist.'"[2] In "Fünf Thesen zur Mikrotonalität," Haas expresses the view that the overtone series is no more important than any other organizing principle of pitch, stating that it is "exactly as artificial as any other musical material,"[3] a statement that puts him in line with American composer James Tenney who has said: "I'm not using the harmonic series to imitate something else… It's a unifying structure."[4] Haas's overtone-based harmony reflects this view by employing the mathematically-perfect series (f1, f2, f3, etc., when f = fundamental frequency) rather than real-world spectra (usually determined through a Fast Fourier Transform analysis[5]) or other distorted inharmonic spectra typically preferred by the French spectral composers like Murail and Grisey. In his introduction to a special issue of *Contemporary Music Review* dedicated to spectral music,

[2] Haas (2007a)
[3] Haas (2007b)
[4] Tenney (2008)
[5] Fineberg (2000a, 99)

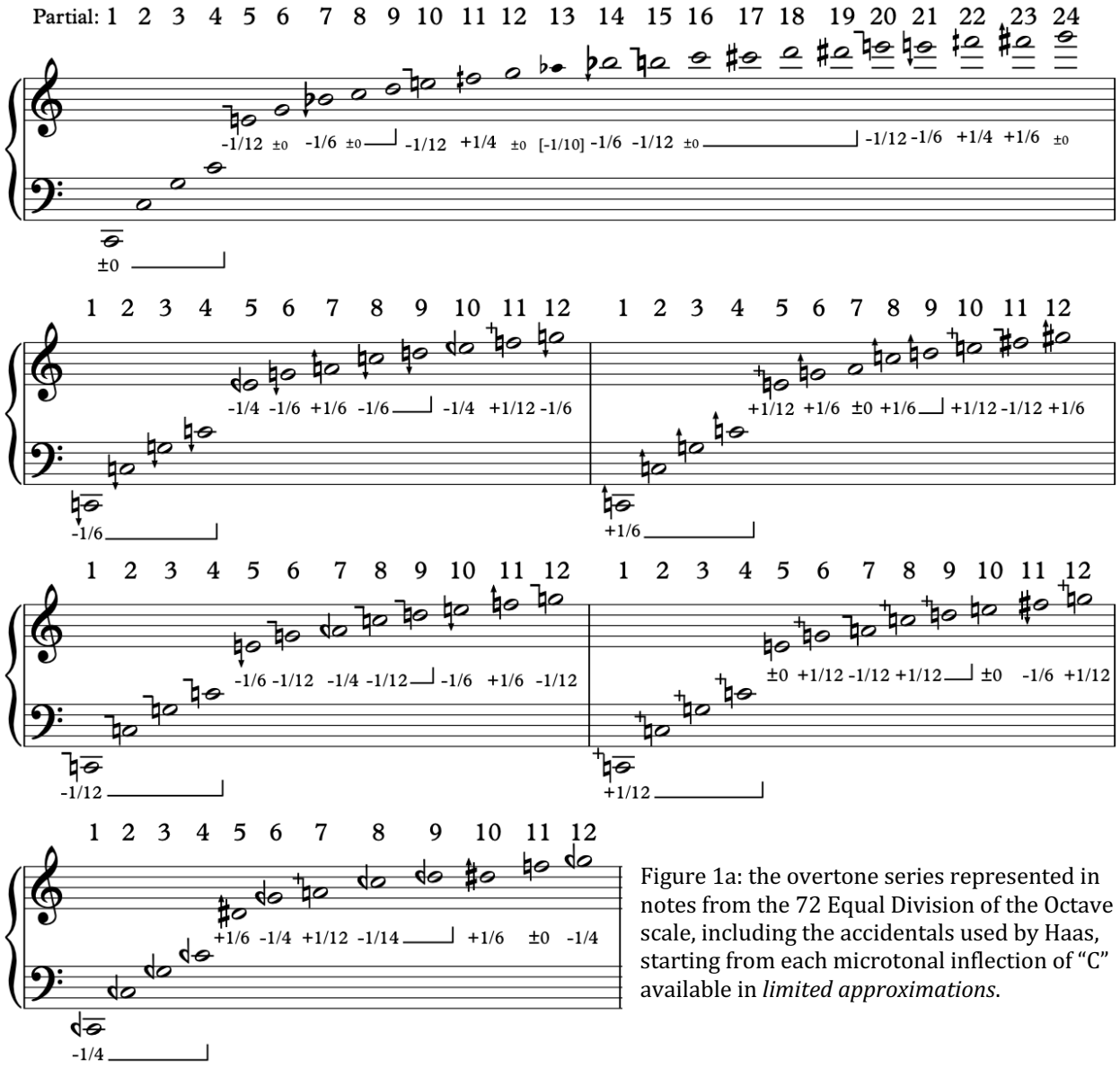



Figure 1a: the overtone series represented in notes from the 72 Equal Division of the Octave scale, including the accidentals used by Haas, starting from each microtonal inflection of "C" available in *limited approximations*.

Joshua Fineberg writes: "Combining and manipulating spectral materials in the same abstract ways in which intervallic materials are treated … does not yield music that I would classify as spectral."[6] The overtone series does not define *limited approximations*, but analyzing Haas's method of incorporating it is important in understanding the work as a whole.

Throughout the work, Haas builds overtone chords with as many as 34 partials,

[6] Fineberg (2000b, 2-3)

| Partial | Overtone class | Approximate Note Name | Cents deviation from 12EDO | 72EDO note (1/12-tone deviation from 12EDO | Piano | JI ratio |
|---|---|---|---|---|---|---|
| 1 | 1 | G | 0 | G | 3rd | 1/1 |
| 2 | 1 | G | 0 | G | 3rd | 2/1 |
| 3 | 3 | D | +2 | D | 3rd | 3/2 |
| 4 | 2 | G | 0 | G | 3rd | 2/1 |
| 5 | 5 | B | –14 | B –1/12 | 4th | 5/4 |
| 6 | 3 | D | +2 | D | 3rd | 3/2 |
| 7 | 7 | F | –31 | F –1/6 | 5th | 7/4 |
| 8 | 1 | G | 0 | G | 3rd | 2/1 |
| 9 | 9 | A | +4 | A | 3rd | 9/8 |
| 10 | 5 | B | –14 | B –1/12 | 4th | 5/4 |
| 11 | 11 | C# | –49 | C +1/4 | 6th | 11/8 |
| 12 | 3 | D | +2 | D | 3rd | 3/2 |
| 13 | 13 | Eb | +41 | | n/a | 13/8 |
| 14 | 7 | F | –31 | F –1/6 | 5th | 7/4 |
| 15 | 15 | F# | –12 | F# –1/12 | 4th | 15/8 |
| 16 | 1 | G | 0 | G | 3rd | 2/1 |
| 17 | 17 | G# | +5 | G# | 3rd | 17/16 |
| 18 | 9 | A | +4 | A | 3rd | 9/8 |
| 19 | 19 | Bb | –2 | Bb | 3rd | 19/16 |
| 20 | 5 | B | –14 | B –1/12 | 4th | 5/4 |
| 21 | 21 | C | –31 | C –1/6 | 5th | 21/16 |
| 22 | 11 | C# | –49 | C +1/4 | 6th | 11/8 |
| 23 | 23 | C# | +28 | C# +1/6 | 1st | 23/16 |
| 24 | 3 | D | +2 | D | 3rd | 3/2 |
| 25 | 25 | Eb | -27 | | n/a | 25/16 |
| 26 | 13 | Eb | +41 | | n/a | 13/8 |
| 27 | 27 | E | +6 | | n/a | 27/16 |
| 28 | 7 | F | -31 | F –1/6 | 5th | 7/4 |
| 29 | 29 | F | +30 | F +1/6 | 1st | 29/16 |
| 30 | 15 | F# | –12 | F# –1/12 | 4th | 15/8 |
| 31 | 31 | F# | +45 | F# +1/4 | n/a | 31/16 |
| 32 | 1 | G | 0 | G | 3rd | 2/1 |
| 33 | 33 | G | +50 | G +1/4 | 6th | 33/32 |
| 34 | 17 | G# | +5 | G# | 3rd | 17/16 |

Figure 1b: the harmonic series on G mapped onto 72 EDO, with approximate just intonation ratios. Grayed-out partials do not appear in *limited approximations.*

using the system of 72 equal divisions of the octave to represent a variety of overtones with considerable accuracy, as shown in Figure 1a and 1b (which is built on the fundamental G to aid with analysis of "Aria I" to be discussed later). All of the pitches used are within five cents of their corresponding overtone. For example, partials 5, 10, and 20 (the pure major 3rd with a $\frac{5}{4}$ ratio) is 14 cents flat from its 12-tone equal-tempered counterpart, only a 2-cent difference from the available twelfth-tone, which is equivalent to $16\frac{2}{3}$ cents. Likewise,

partials 7, 14, and 21 are 31 cents lower than 12EDO and played here as sixth-tones, $33\frac{1}{3}$ cents flat. Overtones that cannot be approximated to within 5 cents are omitted. These include the 13th and 26th partials (13/8 or 841¢); the 27th partial (27/16 or 906¢); and the 31st (31/16 or 1145¢). Also excluded, despite being almost exactly a 1/6th-tone, is the 29th partial (29/16 or 1030¢), which is a prime harmonic (as are 31, and 13), most likely omitted in order to maintain a prime limit of 19. Figure 1 also indicates each partial's "overtone class," which includes all overtones related by octave transposition, represented by a given overtone's lowest multiple of two. For instance, the 26th partial has an overtone class of 13 and all notes related by octave transposition to a given fundamental have an overtone class of 1. This is useful for identifying new pitch classes in the harmonic series at a glance.

Haas cites two other especially important influences on his harmonic language: the atonal language of the Second Viennese School, especially Anton Webern, and the extensive quarter-tone theories of Ivan Wyschnegradsky, both of whom emphasize the tritone and major seventh in their respective harmonic lexicons. Wyschnegradsky, who expounded his principles in "Manual of Quarter-tone Harmony," first published in 1938, composed pieces employing quarter-, third-, sixth-, eighth-, and even twelfth-tones, among other equal divisions of the octave.[7] The combination and juxtaposition of elements from all these styles is the driving force behind *limited approximations*.

[7] Wyschnegradsky (2017), Skinner (2006, 144-180)

Development through pitch inflection

Regarding the potential for 1/12th-tones as non-harmonic embellishments, Haas wrote in his program note: "The twelfth-tone is so small that it is no longer heard as an interval, but rather as the shading of a single note. A single tone played by a romantic orchestra was a wider frequency. The aural effect of a scale in twelfth-tone intervals is thus similar to a glissando."[8] This approach is compounded by 'shading' notes to the point of creating microtonal clusters, which are often invigorated by trills, tremolos, and portamenti in the strings, as heard the opening texture of the piece, mm. 1-21. An example of this shading effect in the pianos can be heard starting in m.167, where the perfect fifth C-G is repeated and stretched across multiple octaves: starting with the regularly-tuned 3rd piano, the 2nd piano (+1/12th-tone) enters with the dyad in higher octaves followed by the 4th piano (-1/12th-tone) in lower octaves. This process continues, expanding the range of the distorted fifth to an entire half-step (from ¼-tone flat up to ¼-tone sharp) stretched across six octaves (notated C-$G_2$ to C-$G_7$). The different versions or 'shadings' of this dyad are emphasized by the standard-tuned C–G fifth sustained by the strings in every available octave ($C_1$ to $G_7$), which alternate and overlap with passages of the piano's repeated chords, creating strong dissonances with five of the six pianos. The constantly shifting, persistent articulations of the pianos are made to sound still more unstable because the tempo gradually accelerates through every phrase, creating momentum that is juxtaposed with the understated, static strings.

After the gesture is repeated several more times, the *accelerando* becomes continuous and the pitch inflection is stretched even further, down to B–F# in m.186,

[8] Haas (2010b)

before the pianos begin a descent which will last to the end of the section (m.196). The descent, a glissando in twelfth-tones, covers nearly a perfect twelfth, from $C_2$ (m.182) to A-quarter $flat_0$ (m.196). Being the lowest note on any of the six pianos, it marks the bottom of the glissando, though the low strings, which join the pianos by performing portamenti with trills (as in the opening of the work) in parallel with the pianos from m.192-196, finish the gesture on a C-$sharp_0$ thanks to the 5-string contrabasses.

When the *accelerando* and descent become constant in m.182 (a musical idea reminiscent of the striking Shepard-Risset glissando or "continuous Risset scale" effect that concludes Haas's work *in vain* from 2000[9]), the static C–G dyad in the strings is replaced by

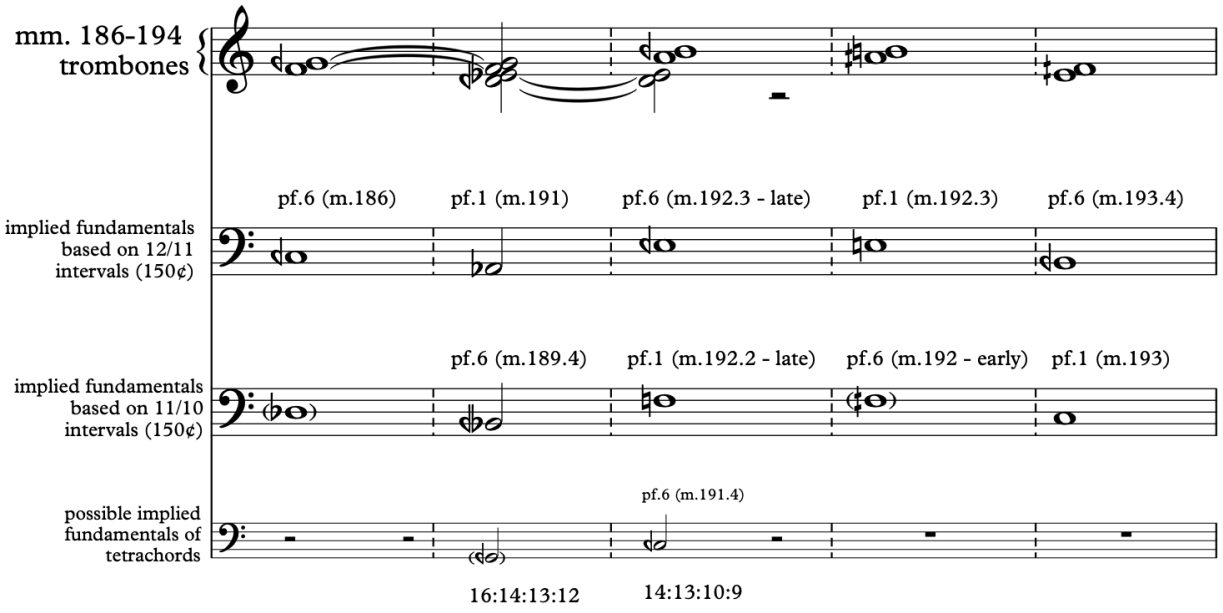



Figure 2: trombone dyads from mm. 186-194 of *limited approximations* with their implied fundamentals, including where (measure.beat) the implied fundamentals occur in the pianos.

the quasi-fanfare entrances of the trombones. Always in pairs, the trombones play a series of dyads, voicing the colorful interval of a 'neutral second' or 150¢, starting in m. 186. This interval can be contextualized in the overtone series in several different ways, the simplest

[9] Hasegawa (2015, pp.218-222)

being approximations of 11/10 or 12/11 (the interval between the 10th and 11th partial, or the 11th and 12th partial, respectively). Thus, with each new dyad, several different fundamentals are suggested. Figure 2 shows the implied root notes for each dyad from mm.186–194, and where that root (and fifth) sounds in the pianos.

By the end of the passage, the pianos are descending at the rate of about a half-step per beat, making it possible to find both potential fundamentals during the duration of a given dyad. For the third dyad, however, both fundamentals (A and B-quarter flat) sound about one bar late — this could be understood as a displacement, but since this dyad overlaps with the one previous (D-quarter flat, E-flat), it is possible to analyze all four notes as part of a single spectrum. The four notes might be heard as a 14:13:10:9 tetrachord, suggesting a fundamental of C-quarter flat, and despite the requirement to temper the intervals by as much as 30¢ to fit them into a C-quarter flat spectrum, it might be valuable for two reasons: first, the fundamental is sounded in the piano at exactly the right moment (m.191, beat 4), and second, the multiple versions of C-quarter flat in the pianos blur the intonation of the fundamental, just as the trombones' tetrachord blurs the spectrum above it. The ambiguity of this passage is emblematic of the way sonorities in *limited approximations* regularly amalgamate multiple harmonies and harmonic systems. An amazing effect of this dramatic music is the aural discord as the timbre of the piano seems to slide gracefully downward, while the trombones, one of the orchestra's most adept executer's of *portamento*, remain stubbornly and majestically motionless.

Development through juxtaposition

To contrast with overtone-based sonorities Haas draws on influences from Wyschnegradsky and the Second Viennese School, which can be heard in his frequent use

of a 12EDO sonority constructed by alternating tritones and perfect fourths. The tritone epitomizes equal temperament because it divides the octave perfectly in half, or 600 cents. The frequencies of the tritone are represented by the ratio $\frac{\sqrt{2}}{1}$, an irrational number that cannot be found in the overtone series. Compare this to $\frac{7}{5}$, the rational interval formed between the 5th and 7th partial (583¢) or to $\frac{17}{12}$ (603¢). Despite resembling an equal division very closely, even these intervals would distort the octave dramatically if stacked on top of one another. For example: 583¢ + 583¢ = 1166¢, a full 34¢ (or 1/6th-tone) shy of a perfect octave. Or combine four intervals of 603¢ and you end up with 2412¢, or two octaves plus 12¢, nearly a 1/12th-tone. The $\frac{\sqrt{2}}{1}$ tritone is, therefore, uniquely able to represent equal temperament as a contrast to the simple ratios of just intonation and the overtone series. Although common-practice tonal music may be the most ubiquitous style to utilize a 12EDO tuning system, it is by no means representative of it. Common practice tonal music in Europe developed from just intonation systems to include a variety of meantone and well-tempered tunings before equal temperament was theorized, and still longer before a true equal division of the octave was executed accurately.[10]

The first appearance of any clear 12EDO harmony occurs at the entrance of four clarinets and four trombones in measure 21. It is the first appearance of any woodwinds or brass, the first instance of a stable pitch or sonority, and a perfect exemplification of the role of the equal-tempered tritone in this work. As the pianos fade out, the passage begins with an F-sharp$_2$, then stacks tritones up to C$_6$, covering exactly three and a half octaves in a gesture that gently evokes sweeping through an overtone series. The trombones and

[10] Partch (1974), Duffin (2007)

clarinets (doubtlessly chosen for their aptitude, rare among wind instruments, for *portamento*) then slide to the next sonority, a dense alternation of tritones and perfect fourths from G-sharp$_2$ to B$_6$ (Figure 3a). This pattern of intervals (hereafter referred to as the tritone-fourth chord) is chosen because of its ability to exhausts all 12 chromatic notes of the 12EDO scale before repeating any, the same property that attracted atonal composers in the 20$^{th}$ century. In measure 23, the clarinets and trombones slide to another stack of tritones (G – C#) to conclude the passage. By ending a semitone higher than where the passage began, the inherent limitations of 12EDO are emphasized; when heard next to the constant microtonal saturation from the pianos and strings, a half-step sounds like a very large interval.

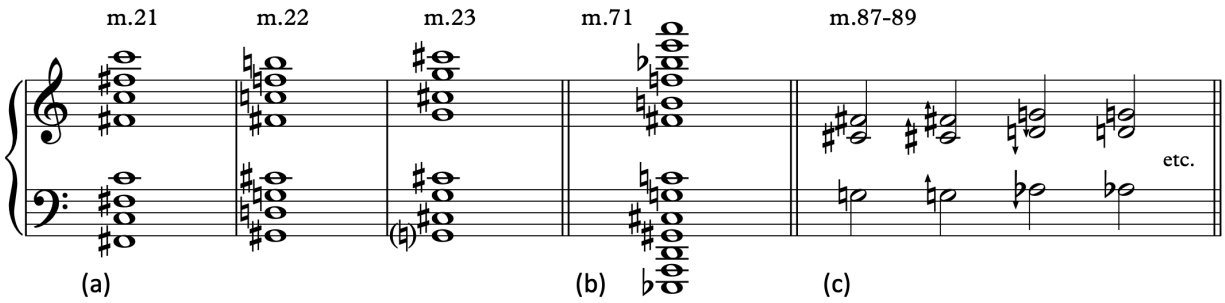



Figure 3: (a) clarinets and trombones in measures 21-23, voicing equal tempered sonorities based on tritones and perfect fourths (b) strings in measure 71, voicing the first complete 12-tone version of the tritone-perfect fourth sonority, with A doubled. (c) trichords derived from the tritone-fourth sonority, beginning in measure 87 in winds and brass, intoned at succesive 1/6th tones.

The first time the tritone-fourth chord is fully realized is in measure 71 where it is built on E-flat$_1$ and ascends, tritone followed by perfect fourth, up to A$_6$, so that the 12EDO aggregate is represented (Fig. 3b). Notes are doubled only as the chord is expanded past 12 notes, most commonly to 13. In this configuration, A is the second and thirteenth note from the bass, doubled at A$_1$ and A$_6$. The strengthening of A-natural is established just prior to this (mm.46-52) when A-natural emerges in the low strings, following closely by E-natural

in the violins, providing clarity to the microtonal clusters in the pianos, which are emphasizing notes from an A overtone chord. Particularly in mm. 49-50, at the peak of the violins' crescendo, briefly reinforced by some winds, the careful dynamic markings in the pianos reveal that only the 3rd piano (±0) is marked *forte* along with the strings, while the other five pianos continue to play *pianissimo*, providing a fog around the regularly-tuned A–E fifth. As the strings fade out, the A fundamental is shifted and obscured when $A^{+1/12}$ then $A^{+1/6}$ emerge from the texture, followed by $E^{-1/12}$ and $E^{-1/6}$, collapsing the perfect fifth. This pitch proves important when, at the end of the piece, the A fundamental is used to build an unobscured OTC up to the 24th partial, concluding the entire work. In music which avoids and defies traditional tonal centers (as a function of common-practice tonal harmony) a 'pitch center' is often established through repetition and registral accent, as Haas does here. This method is *inherent* in the spectralist approach to harmony because a fundamental is emphasized aurally as the bass note and is repeated in the overtone series more than any other pitch.

A strong instance of 12EDO harmony being pitted against overtone chords very explicitly occurs in measures 67-83. An overtone chord on E-flat$^{-1/6}$ is voiced up to the 11th partial in the pianos with the first two partials (octave E-flat$^{-1/6}$) supported in the contrabasses and cellos (m. 67). This alternates and overlaps with the 12EDO tritone-fourth sonority discussed above (m.71), based on a regularly-tuned E-fat, making the bass note of each harmony only 33 cents apart. Furthermore, an overtone chord based on a pitch 1/6th-tone flat produces no partials that are approximated to notes in the regular 12EDO scale until the 23rd partial, the only twelfth tone for which this is the case (see Figure 1a). Thus, no common tones or 12EDO intervallic relationships are present between

the two harmonies, making the contrast as stark and deliberate as possible.

This passage draws into focus two important axes that serve as points of juxtaposition throughout the work. First is harmony: 12-tone equal-tempered against acoustic or overtone-derived. This axis is, in part, a manifestation of the antagonistic relationship between 12EDO and harmonic spectra, but it is important to note that the differences lie in the internal relationship between the harmonic structures rather than tuning; a 12-tone chromatic chord may be constructed from the standard 12EDO scale at A440hz or $A^{+1/6}$ or $A^{-1/12}$ or any other microtonal inflection and still belong to the world of 12EDO. The second axis, the inflection of pitch, expands the idea that the $1/12^{th}$-tone can be used to shade or color a single note.

By measure 71, two fully expanded harmonic entities coexist, each one falling neatly into opposing categories as outlined by the first axis above. The contrast is illuminated through orchestration: the overtone chord is voiced in the pianos and lowest strings, while the upper strings voice the 12-tone chord. The second of these axes, intonation, illustrates what Haas calls *Klangspaltung*, the 'sound-splitting' effect of closely spaced microtones around a single pitch.[11] This occurs between the two fundamentals, but also in the upper voices. For example, $G^{-1/4}$ in the $6^{th}$ piano clashes strongly with the G-natural in the $4^{th}$ viola and F-sharp in the $8^{th}$ violin II.

The score also indicates open strings wherever possible, a technique that centers the pitch by eliminating vibrato and brightens the sound in order to highlight the natural overtones of the strings. This results in additional instances of *Klanspaltung* between strong overtones and sounding pitches. The first six partials (tempered to 72EDO) of the

[11] Haas (2007, p.10), Hasegawa (2015, pp.205-206)

open A in the 3rd contrabass are $A_1$, $A_2$, $E_3$, $A_3$, C-sharp$_4^{-1/6}$, and $E_3$; the second and fifth partial particularly clash with B-flat$^{-1/6}$ in the 5th piano (a 67-cent discrepancy) and C$^{+1/6}$ in the 1st piano (a 50-cent discrepancy), respectively, as shown in Figure 4.

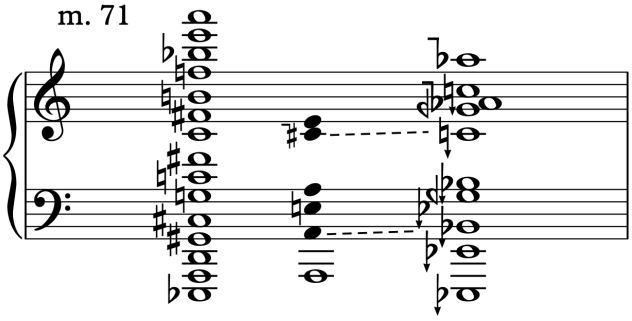



Figure 4: the extended tritone-fourth chord sounding in the upper strings, the implied overtone chord above the contrabass's open A string, and the E-flat$^{-1/6}$ as voiced in the pianos in measure 71. Two examples of *Klangsplatung* are shown with dotted lines.

As is typical in this piece, the clarity of each harmony that was established in m. 71 is severely compromised going forward. The dynamics of the two harmonies alternate radically. When the 12-tone chord comes to the fore (mm. 71-74 and 78-80), the pianos (which voice the OTC, exclusively) fade out completely. Throughout these sixteen bars, the registral division in the strings is obscured and the E-flat$^{-1/6}$ overtone chord is amalgamated with the E-flat 12-tone chord, destroying the clear contrast and merging the two sonorities into one. By m. 80 some strings begin to glissando and add very wide vibrato (marked "ca. 3/8-Ton," equivalent to 75 cents on either side of the written pitch) to further obscure the sonorities. Also in m. 80, the 12-tone chord emerges in the winds and brass, competing with the pianos' final tremolo assertion of the E-flat$^{-1/6}$ overtone chord (mm. 81-82). The fresh timbre of the winds and brass, the first to voice the 12-tone sonority in m. 21, marks the beginning of a transition and facilitates the 3rd piano entrance in measure 84.

Dramatically different from the surrounding textures, this is an unmistakably important entrance that momentarily takes over the texture in mm. 84-85. Repeated ascending hexachords outline the 12-tone chord that is simultaneously fading out in the

winds. Several things are happening here: this is the first time the piano timbre articulates the 12-tone chord. It also signals a role-reversal because the piano is now the stable harmonic entity, while the strings' wide vibrato and glissandi provide the obfuscation of pitch previously performed by the ensemble of pianos. This moment is also striking because of the new rhythmic ideas. The formal significance of this motive will be discussed below.

Underneath this isolated statement in the piano and immediately following it, the overall texture is a string-dominated amorphous sound almost identical to the beginning of the piece. In measure 87, however, the winds and brass begin playing trichords derived from the 12-tone chord, alternating tritones and perfect fourths. As shown in Figure 3(c), these successive trichords are intoned sixth-tones apart with the lowest notes of the first five as follows: G, $G^{+1/6}$, A-flat$^{-1/6}$, A-flat, $A^{-1/6}$. This microtonal ascent continues, recalling and developing the 1/6$^{th}$-tone relationship between the 12EDO chord and overtone chord from mm. 67-80 (E-flat and E-flat$^{-1/6}$, respectively). Here, Hass has isolated the expressive axis of 'inflection' introduced above by using the 1/12$^{th}$-tone (72EDO) system to explore microtonal inflections of a paradigmatic 12EDO sonority, an idea that will be returned to at the conclusion of the entire piece.

Later in the work, the same technique of juxtaposition is used in reverse: in measures 218 to 234, two overtone chords with fundamentals an equal-tempered tritone away, alternate and combine. The strings voice a clear E-flat overtone chord while the winds voice a clear A-natural overtone chord. This music (starting approximately 16 minutes into the work) marks the start of a section which is to have a much slower overall harmonic motion. Despite the apparent disparity between these two sonorities, the

transitions from one to another are remarkably smooth, a feat made possible because of a remarkable use of pitch-shading in the 4th and 5th pianos, which play throughout. Each piano part is a constant reminder of the tritone: the 4th piano plays C-sharp$_4^{-1/12}$ and G$_4^{-1/12}$, the 5th partial (or $\frac{5}{4}$ just intonation major third) of A and E-flat, respectively; the 5th piano plays D-flat$_4^{-1/6}$ and G$_4^{-1/6}$, the 7th partial of E-flat and A, respectively. There are a lot of connections to be made here: first, each piano independently highlights the tritone; second, both overtone chords are supported by the two $\frac{7}{5}$ intervals (the interval between the 5th and 7th partials, which is 583 cents) voiced across the pianos; and third, the intense and continuous *Klangspaltung* that occurs between these two dyads which are only 1/12th-tone (16.6 cents) apart. The pianos effectively widen each of these two pitches, just as an instrument playing with vibrato would. Our ears 'average' the two versions, hearing a dyad exactly in the middle of the two pianos, at 1/8th-tone flat, or 25 cents. The irregular and polyrhythmic repetition of the four notes, such as 6 against 5, 7 against 6, and 8 against 7, contributes to the sense of artificial vibrato. This remarkable shading of the pitch creates a harmonic thread between the two overtone chords, imaginatively connecting two isolated sonorities through common-tones.

The shading of this dyad continues into m. 235 when it is expanded by adding C-sharp (3rd piano) and G$^{-1/4}$ (6th piano). These two notes widen the pitch space even further and will also serve as the 1st and 11th partial of a C-sharp overtone chord. In mm. 236 and 237, the C-sharp overtone chord fades in, sweeping slowly upwards in the strings while the 4th and 5th pianos fade out. This chord in the stings, supported by the 3rd and 6th piano with frequent entrances of the winds and brass, continues until m. 246.

Another slow transition occurs from mm. 243 to 249. First, the G$^{-1/6}$ and C-sharp$^{-1/12}$

reappear in the 5th and 4th pianos, respectively, obscuring the pitch, but also providing a link to an A overtone chord (7th and 5th partials), which promptly appears as a flourish in the winds and brass. As the C-sharp overtone chord in the strings fades out completely, the 1st piano enters on a tremolo between C-sharp$^{+1/6}$ and F-sharp$^{+1/6}$ — a further expansion of the C-sharp upwards and the G downwards — which prepares the entrance of the strings in measure 250 on an F-sharp$^{+1/6}$ overtone chord. In measure 252 the compression of the dyad continues when and F-sharp$^{+/-0}$ (3rd piano) and a D$^{-1/6}$ (5th piano) enter to prepare a rich E overtone chord, establishing the 9th and 7th partials, respectively. In measures 257 to 261, the compression is taken one step further when D$^{+/-0}$ (3rd piano) and a F-sharp$^{-1/12}$ (5th piano) enter, becoming the 1st and 5th partials of a new D overtone chord. The E and D overtone chords alternate, a relationship that once again highlights a 1/6-tone via the D-natural to D$^{-1/6}$ (7th partial of E). By juxtaposing two overtone sonorities with roots an equal-tempered minor seventh apart, Haas highlights the differences between the two systems of harmony while combining them, just as before when overtone chords where separated by an equal-tempered tritone (mm. 218-234).

This passage concludes with the emergence of a striking, pristine D overtone chord in measure 268, brightened by *estremamente sul ponticello* in the basses, while the celli play *ordinaro* in unison to provide a strong foundation. Above, solo strings (two violins, viola, two celli) play D's and A's, oscillating between *sul ponticello* and *ordinaro* at different rates to create a shimmering quality. A solo flute plays the 7th partial (C$^{-1/6}$) and two solo horns play the 8th and 10th partials (D and F-sharp$^{-1/12}$), lightly establishing the sonority of an overtone chord.

Structure defined through motive

While this paper focuses on harmonic analysis and its implications, it is important to notice how the rhythmic and gestural motives in this work reinforce, and even define, the structure. There are actually very few clearly metric structures in *limited approximations*. Although the rhythmic information is often very dense, it serves a primarily atmospheric purpose, keeping a low profile by dictating staggered entrances (mm. 79-83, winds), organizing sequential waves of crescendos and diminuendos (mm. 36-37, strings), or weaving a fabric of polyrhythmic repeated notes (mm. 31-32, pianos). The last of these may be considered a rhythmic analogy for the blurring of pitch achieved through $1/12^{th}$-tone shading, trills, glissandi, and combinations thereof, that is so important to the piece.

Several gestures serve as structural landmarks. The first, *a*, is a low rumble produced by simultaneous ascending chromatic scales starting from each piano's lowest note, $A_0$. completely saturating the 72EDO pitch space (mm.18-20). Reminiscent of a bass drum or thunder sheet, this microtonal cluster, is a chaotic way of completely saturating the 72EDO pitch space without favoring any type of harmony or sonority. The second gestural motive to stand out, *b*, first occurs in measures 84-86, piano 3. In its first, isolated iteration, discussed above, the repeated, ascending hexachords outline the 12-tone chord (alternating tritones and perfect fourths) that kicks off the exploration of this sonority by inflecting a representative trichord in $1/6^{th}$ tones. This simple rhythmic gesture of repeated notes becomes central to an entire section of music starting in measure 167 when the rhythm stabilizes to constant eighth notes as a C–G perfect fifth is inflected microtonally, as detailed above. These gestures are recognized as related because their regular, repeating rhythm is very distinct amidst the amorphic rhythmic language of the piece. They are also

linked by being associated with the same mode of development through microtonal inflection.

Immediately following the section built on motive *b*, a third motivic gesture, *c*, comes to prominence (mm. 195-211). Appearing in the winds (mm. 195-196), motive *c* is related to *a* in that it is an ascending gesture with indistinct pitch and a busy surface. There are two main differences, however: first, *c* is presented in short, quick bursts, while *a* lingers and fades out naturally; and second, *c* is most accurately described as an arpeggio, while *a* is a scale (although in this initial presentation *c* is an overtone arpeggio, notated as a glissando in the horns, which actually manifests itself as a microtonal scale in the upper partials). Motive *c* appears again in the horns in mm. 264 to 266, which sets up a very significant event, the transfer of the motive to the pianos (mm. 278- 295). When appearing in the pianos, the range of motive *c* is greatly extended in both directions and is necessarily divided between the keyboards to accurately represent the overtone chords, which are voiced up to the 34th partial, the farthest-reaching harmonic series in the work.

## The 'Arias'

Approaching the end of the work, Haas clearly labels 'aria I' (mm. 342-346) and 'aria II' (mm. 391-399) in the score. Both of these striking sections inhabit sound worlds unique to the composition. The harmonic foundation for 'aria I' is a G-natural overtone chord with no inflection of pitch or other disruptions. The five-measure long aria is characterized by a rising unhurried melody shaped exclusively from upper partials of G-natural (see Figure 5). Each new pitch, articulated in the pianos, is foreshadowed and sustained in either the strings or winds. When the gradual ascent reaches the 23rd partial, the strings (divided into 17 parts) are left sustaining a very full-bodied overtone chord. The aria mimics a

traditional melodic shape by rising in waves to the penultimate note before resolving downward by a step (the 'step' in this case is the interval $\frac{23}{22}$ or about 77¢). Analyzing the relative tension created by each overtone confirms that the melody can be broken down into two elided phrases marked by resolutions from complex to simple overtone classes. The first phrase begins on the 7th partial and ends a perfect octave higher, on the 14th. Climbing upward through overtones with simple ratios (7, 10, 9), the first phrase reaches its apex at the 15th partial, which drops to the 11th partial to bracket the note of resolution, the 14th partial. The stepwise descent from partial 15 to 14 (about 119¢) is an audible resolution, reinforced by the long duration of the final note. As the second phrase continues to rise in register, it passes through higher octaves of lower partials like 14 and 20 (overtone classes 7 and 5, respectively) before reaching the apex on two complex harmonics, 21 and 23, which straddle the final 22nd partial (overtone class 11) and provide another stepwise resolution down from a complex to simple overtone class. Since the second phrase concludes with a prime harmonic (thus with a more complex ratio, making it more dissonant) it is fair to say that it is the stronger cadence. Therefore, it is reasonable to conclude that Haas has established an analogue to a traditional period phrase structure with an antecedent and consequent structure. By using overtones in this way, Haas balances the constant registral ascent necessary for building an overtone chord from the fundamental, while simultaneously creating tension and release in the melody.

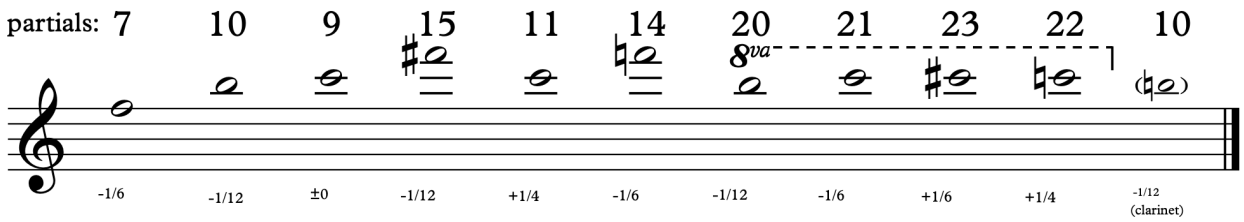



Figure 6: the pitch inflection of the overtone chord throughout the intermezzo, represented by the fundamental moving in 1/12-tone increments.

In measure 347, the pianos blur the G fundamental with microtonal variations of the first several partials, though still retain the proper tuning of the G-natural OTC for repeated *c*'-like ascending arpeggios, while the strings sustain continue to sustain the 23-partial G-natural overtone chord unwaveringly. This antagonistic relationship continues through the 'intermezzo' (mm. 359-373), recalling measure 167, when the same technique was used to ornament a C–G dyad. The 'intermezzo' locks in the microtonal variation by actually inflecting the entire OTC in parallel, including the fundamental, as shown in Figure 6. This section is audibly related to motive *b* due to the eighth note rhythm, which is divided into uneven phrase lengths, and the development through inflection that accompanies it.

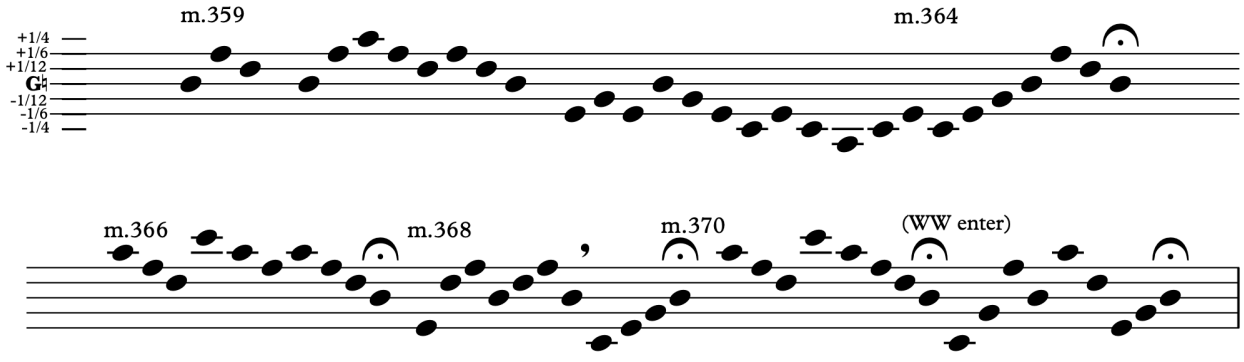



Figure 5: the melody of "Aria I" shown with pitch inflections below the staff and indicating what partial of the G overtone chord each melody note corresponds to.

Another bridge (mm. 376-390) connects the intermezzo to 'aria II.' Establishing a tapestry of polyrhythmic repeated notes, the pianos voice a G-natural overtone chord that slowly shifts upward by 1/12$^{th}$-tones, a measured glissando that traverses a minor third

plus a 1/12th-tone (equivalent to a $\frac{6}{5}$ just intonation minor third, or 316¢). The first violins (divisi à10) all fade out leaving only the first seven partials of the standard-tuned G-natural overtone, lightly reinforced by some winds and brass. The rate of the pianos' ascent increases until the sound is completely chaotic; tremolos, glissandi, and trills in the strings and winds replace the standard-tuned G overtone chord, totally obscuring the harmony as the whole orchestra swells to *ff*. An abrupt cut-off reveals dense microtonal clusters being sustained in the lowest register of the pianos, the ringing-out of six overlapping chromatic scales, motive *a.*

Emerging from this cluster, 'aria II' (mm. 391-399) contrasts with 'aria I' by taking a quasi-chorale approach to melody. The pianos voice a series of 23-partial overtone chords whose fundamentals move in an unpredictable succession of twelfth-tones. Since every chord is really an expansion or realization of the fundamental, this aria turns the traditional chorale on its head by drawing the ear to the lowest sounding note as the melody. The constant shifting of the fundamental is also anti-spectalist by never allowing any one harmonic spectra to establish itself. Like 'aria I,' this melody is constantly rising, but here the ascent is much more dramatic because as the fundamental rises, the upper partials disappear as they become too high for the pianos to voice them, until the final note of the melody is only the first three partials (an octave and 12th). Underneath the aria's apparent microtonal saturation, the strings fade in playing *flautato* trilled glissandi.

Finale and conclusion

In the final bars (mm.400-431), which are marked 'finale' in the score, a 24-partial A-natural overtone chord emerges in the strings. This harmony serves as a backdrop for interjections in the pianos, playing versions of the 12-tone chord that are blurred in two

ways: pitch and rhythm. Each piano is playing the arpeggio at its own twelfth-tone inflection. Rhythmically, each piano starts apart from the others, plays the arpeggio at a different speed, or both. The effect of this differentiation is to create a single ascending gesture, motive *c'*. Although the time interval between gestures is varied and unpredictable, the rhythmic content of each one is identical, which also aids in the perception of a single unit or motive. Variation comes from twelfth-tone inflections, subtle dynamic shifts, and reordering of the 12-tone chord itself (whether the first interval above the bass is a tritone or perfect fourth).

The work concludes with the complete A-natural OTC and the final version of the microtonally-inflected tritone-fourth chord in the pianos ringing out together, fading to nothing. Several structural tensions are resolved through this gesture. First, the hyper-chromatic scalar presentation of pitch material that the pianos produce at the beginning of the work, (motive *a*) is transformed by carefully structuring the complete saturation of 72EDO pitch-space as a series of tritone-fourth arpeggios (motive *c'*). This links motives *a* and *c*, while also using inflection to voice the tritone fourth (12-tone) chord in its most complete form within the context of this work: as a 72-tone sonority. Furthermore, the rich but chaotic string music that opens the work is in contrast to the crystal clear A overtone chord for the final moments of the work, suggesting an arrival, or coming to rest.

Overall, *limited approximations* thematizes the harmonic relationships of 12EDO and 72EDO as epitomized by the 12-tone chord and the overtone chord, respectively. Tension is created by juxtaposing these two harmonic systems and by imposing aspects of one onto the other, such as presenting overtone chords with roots an equal-tempered tritone or minor seventh away, and the opposite, inflecting the 12-tone sonority by $1/6^{th}$- and $1/12^{th}$-

tones. Further analysis of this work should investigate the effects of orchestration on the harmony, particularly in regards to instrumental synthesis. More work could also be done in exploring the relationships of pitch centers throughout. Four of the five main pitch centers (not accounting for microtonal inflections) are C, E-flat, F-sharp, and A, which form a cycle of minor thirds. The fifth significant pitch center, G, is unique to the "aria I" and "intermezzo" sections. There are many subtle relationships waiting to be revealed.

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